Engineering Approximation of Beam Flutter

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A technique is presented for rapid construction of approximate flutter boundaries for a beam without access to detailed structural information. The method is a useful preliminary design tool which permits easy evaluation of the effects of design changes thereby facilitating finding the means to eliminate aeroelastic instability, should it occur. An example is presented from a recent analysis of the dynamic aeroelastic stability of Space Shuttle protuberances.

Nomenclature

A = total cross-sectional area
A_M = fraction of A made of metal
b = span
c = beam width
F = force
f =
h = beam height
l = sectional lift, coefficient $c_l = l/(\rho V^2/2)c$
M = Mach number
$\tilde{m}$ = generalized mass
m_p = sectional pitching moment, coefficient $c_m = m_p/(\rho V^2/2)c^2$
n = sectional normal force, coefficient $c_n = n/(\rho V^2/2)c$
$P(t)$ = generalized force
$q(t)$ = normalized coordinate
r_I = radius of torsional inertia
T = torsional moment
t = time
V = cross flow velocity
$\bar{V}$ = reduced velocity, $\bar{V} = V/fh$
y = spanwise coordinate
z = vertical coordinate (Fig. 1)
Δ = increment
ζ_0 = structural damping, fraction of critical
$\zeta_a + \zeta_s$ = aerodynamic damping, fraction of critical
η = dimensionless y coordinate, Eq. (10)
θ = perturbation in pitch and torsion (Fig. 1)
κ = dimensionless radius of torsional inertia, $\kappa = r_I/(c/2)$
ρ = air density
$\tilde{\rho}_B$ = effective density of beam
ρ_M = density of metal used to construct the beam box
ρ_{TPC} = density of thermo-protection cover on beam
ϕ = normalized modal deflection
ω = angular frequency, $\omega = 2\pi f$

Subscripts

a = attached flow
b = bending
B = beam
E = estimate
M = metal
s = separated flow
TPC = thermo-protection cover
t = torsion or time
V = cross flow component
∞ = Shuttle reference conditions

Superscript

i = separation-induced, e.g., $\Delta^i C_N$ = separation-induced normal force

Derivative Symbols

$$c_{m\alpha} = \frac{\partial c_m}{\partial \alpha}$$

$$\dot{\alpha} = \frac{\partial \alpha}{\partial t} : \ddot{q} = \frac{\partial^2 q}{\partial t^2}$$

$$c_{mq} = \frac{\partial c_m}{\partial (cq/V)}$$

$$c_{m\dot{\theta}} = \frac{\partial c_m}{\partial (c\dot{\theta}/V)} : c_{m\dot{z}} = \frac{\partial c_m}{\partial (\dot{z}/V)}$$

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Introduction

AEROELASTIC stability analysis of elastic vehicle dynamics involves the comparison of structural and aerodynamic damping components of roughly equal size. Quite often the design is in a state of flux and the aeroelastic analysis has to be repeated continually to reflect the effects of changing structural characteristics. In case of indicated aeroelastic instability, the structural dynamicist needs guidance in determining the needed design changes. The present paper describes a technique that provides the flutter boundaries for one-degree-of-freedom oscillations in bending or torsion in terms of structural characteristics without the need for repeated computations of the effects of structural changes. The technique is accurate enough for preliminary design and facilitates the quick assessment of the effect of design changes on the aeroelastic stability. Due to its simplicity, the technique also helps the designer determine what changes will be needed to avoid beam flutter. The example chosen to illustrate the use of the technique is a recently performed analysis of the aeroelastic stability of the cable trays on the Space Shuttle external tank.[1,2]

Analysis

The dynamics of the beam are analyzed for oscillations in bending and torsion. The critical mechanisms for single-degree-of-freedom flutter in either bending or torsion are described in detail, providing the information needed for a multi-degree-of-freedom analysis.

The equations of motion can be written in the following form using standard notations

$$\bar{m}[\ddot{q}(t)+2\zeta_0\omega\dot{q}(t)+\omega^2 q(t)]=P(t) \tag{1}$$

The generalized force $P(t)$ is given by the virtual work done by the aerodynamic forces and moments (if W is the work done, $P=\partial W/\partial q$).

For the bending degrees of freedom $P(t)$ is

$$P(t)=\int \frac{\mathrm{d}F}{\mathrm{d}y}\phi_b(y)\mathrm{d}y \tag{2}$$

where $\mathrm{d}F/\mathrm{d}y$ is the normal force per unit span and $\phi_b(y)$ the normalized bending mode. For the torsional degrees-of-freedom $P(t)$ becomes

$$P(t)=\int \frac{\mathrm{d}T}{\mathrm{d}y}\phi_t(y)\mathrm{d}y \tag{3}$$

where $\mathrm{d}T/\mathrm{d}y$ is the torsional pitching moment per unit span and $\phi_t(y)$ the normalized mode.

There are three different types of generalized force

$$P(t)=P_s(t)+P_a(t)+P_f(t) \tag{4}$$

While $P_a(t)$ and $P_f(t)$ often can be defined by established methods, $P_s(t)$ can usually not be defined without a special analysis tailored to the problem at hand.

Figure 1 shows the beam in crossflow of velocity V. For small perturbations $\zeta=z/c$ and θ the sectional aerodynamic force and moment induced by separated flow can be expresssed in derivative form as follows.†

$$\frac{\mathrm{d}F_s}{\mathrm{d}y}=L_s=\frac{\rho V^2}{2}c\left[\Delta^i c_{n_\theta}\left(\theta+\frac{\dot{z}}{V}\right)+\Delta^i c_{n_{\dot\theta}}\frac{c\dot{\theta}}{V}\right]$$

$$\frac{\mathrm{d}T_s}{\mathrm{d}y}=M_s=\frac{\rho V^2}{2}c^2\left[\Delta^i c_{m_\theta}\left(\theta+\frac{\dot{z}}{V}\right)+\Delta^i c_{m_{\dot\theta}}\frac{c\dot{\theta}}{V}\right] \tag{5}$$

The corresponding expression for the attached flow derivative is

$$\frac{\mathrm{d}F_a}{\mathrm{d}y}=L_a=\frac{\rho V^2}{2}c\left[c_{n_\theta}\left(\theta+\frac{\dot{z}}{V}\right)+c_{n_{\dot\theta}}\frac{c\dot{\theta}}{V}\right]$$

$$\frac{\mathrm{d}T_a}{\mathrm{d}y}=M_a=\frac{\rho V^2}{2}c^2\left[c_{m_\theta}\left(\theta+\frac{\dot{z}}{V}\right)+c_{m_{\dot\theta}}\frac{c\dot{\theta}}{V}\right] \tag{6}$$

Equations (5) and (6) illustrate the aerodynamic coupling between bending and torsional degrees-of-freedom. The structural coupling between those degrees-of-freedom can be expressed as follows:

$$z=\phi_b(y)q_b(t)+\phi_b^t(y)q_t(t)$$

$$\theta=\phi_t^b(y)q_b(t)+\phi_t(y)q_t(t) \tag{7}$$

As the aerodynamic stiffness is negligible compared to the structural stiffness only the dot-terms ($\dot{z}/V$ and $c\dot{\theta}/V$) in Eqs. (5) and (6) need to be retained.

In order to obtain an assessment of the danger-potential of various aeroelastic mechanisms the beam is first analyzed for one-degree-of-freedom oscillations in either bending or torsion. For pure bending oscillations Eqs. (1-7) give the following results.

†The aerodynamic acceleration derivatives are assumed negligible compared to the structural counterparts. All aerodynamic ground-interference type terms, $c_n(z)$, associated with the height z are also assumed to be negligible.

$$\ddot{q}_b(t)+2\omega_b(\zeta_0+\zeta_a+\zeta_s)\dot{q}_b(t)+\omega_b^2 q_b(t)=[P_f(t)]\bar{m}_b \tag{8}$$

In a region of constant cross flow $\zeta_a+\zeta_s$ is defined as follows.

$$\zeta_a+\zeta_s=\frac{\rho Vc}{4\omega_b\bar{m}_b}(c_{n_\theta}+\Delta^i c_{n_\theta})\int \phi_b^2(y)\mathrm{d}y \tag{9}$$

Assuming that the constant cross flow extends between two beam supports, separated by the distance b, Eq. (9) can be rewritten as

$$\zeta_a+\zeta_s=\frac{\rho Vcb}{4\omega_b\bar{m}_b}(c_{n_\theta}+\Delta^i c_{n_\theta})\int_0^1 \phi_b^2(\eta)\mathrm{d}\eta \tag{10}$$

where $\eta=(y-y_n)/(y_{n+1}-y_n)$ with $y_{n+1}-y_n=b$. For a beam with constant cross section, $c\times h$, the generalized mass is simply

$$\bar{m}_b=\bar{\rho}_B hcb\int_0^1 \phi_b^2(\eta)\mathrm{d}\eta \tag{11}$$

where $\bar{\rho}_B$ is an effective average density. For a uniform beam the mode shapes are well approximated by the following sinusoidal.

$$\phi_b=\phi_0+(1-\phi_0)\sin(n\pi\eta) \tag{12}$$

Combining Eqs. (11) and (12) gives

$$\bar{\rho}_B=\frac{\bar{m}_b}{hcb}\left\{\frac{1-2\phi_0+3\phi_0^2}{2}-\frac{2\phi_0(1-\phi_0)}{n\pi}[(-1)^n-1]\right\} \tag{13a}$$

For the case of pinned modes, $\phi_0=0$, Eq. (13a) becomes

$$\bar{\rho}_B=2\bar{m}_b/hcb \tag{13b}$$

Equations (10) and (11) give the following definition of the aerodynamic damping.

$$\zeta_a+\zeta_s=\frac{1}{8\pi}\frac{\rho}{\bar{\rho}_B}\bar{V}(c_{n\theta}+\Delta^i c_{n_\theta}) \tag{14}$$

where $\bar{V}=V/fh$.

Through the use of Eqs. (13a) and (14) an estimate of the aerodynamic damping can be made without access to detailed structural characteristics. The oscillation frequency is the only structural parameter needed. This is of great benefit, as the structural characteristics have a tendency to change with time, either through updated analysis or new design.

For torsional oscillations the aerodynamic moment derivatives $c_{m_{\dot\theta}}$ and $\Delta^i c_{m_{\dot\theta}}$ take the place of the force derivatives c_{n_θ} and $\Delta^i c_{n_\theta}$; the area c^2 takes the place of the reference length c [see Eq. (5)]; and the following definition of the generalized mass is obtained.

$$\bar{m}_t=\bar{\rho}_B r_I^2 hcb\int_0^1 \phi_t^2(\eta)\mathrm{d}(\eta) \tag{15}$$

Representing ϕ_t the same way as ϕ_b, Eq. (12), the integral in Eq. (15) becomes equal to the expression within brackets in Eq. (13a). $\bar{\rho}_B$ is the effective average density of the beam defined by Eq. (13a) and r_I is the cross-sectional radius of torsional inertia.

$$r_I=\sqrt{\bar{m}_t/\bar{m}_b} \tag{16}$$

Approximate Analysis

Instead of determining $\bar{\rho}_B$ from Eq. (13), an estimate can be obtained as follows.

$$(\bar{\rho}_B)_E=\rho_M\frac{A_M}{A}+\rho_{\mathrm{TPC}}\left(1-\frac{A_M}{A}\right) \tag{17}$$

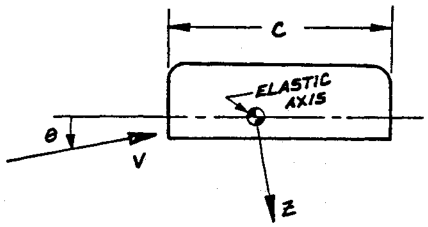

Fig. 1 Definition of coordinates.

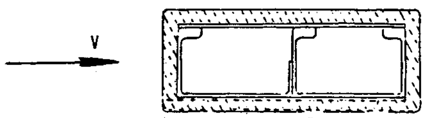

Fig. 2 Structural beam in cross flow.

For a complete spectrum of probable metals, including steel, bronze, and brass, the density ρ_M varies very little.

$$6000 < \rho_M/\rho(0) < 7000 \tag{18}$$

where $\rho(0)$ is the air density at sea level.

A_M is the cross-sectional area of the metal and A is the total cross-sectional (reference) area, $A = ch$. The torsional radius of inertia is also easily determined for a simple metal box cross section.

Flutter Boundary

The flutter boundary is determined by

$$\zeta_0 + \zeta_a + \zeta_s = 0 \tag{19}$$

Combining Eqs. (14) and (20) gives the following flutter boundary for bending oscillations.

$$\frac{\zeta_0 fh}{V} = -\frac{1}{8\pi}\frac{\rho}{\bar{\rho}_B}(c_{n\theta} + \Delta^i c_{n\theta}) \tag{20}$$

The corresponding result for torsional oscillations is

$$\frac{\zeta_0 fh}{V} = \frac{1}{2\pi\kappa^2}\frac{\rho}{\bar{\rho}_B}(c_{m\dot{\theta}} + \Delta^i c_{m\dot{\theta}}) \tag{21}$$

Application to Space Shuttle Cable Tray

In the flutter boundaries defined by Eqs. (20) and (21) the right-hand side is in the approximate formulation independent of structual characteristics. The stability boundary for bending oscillations of the LO_2 cable tray[1,2] is shown in Fig. 3. The flutter boundary obtained using the approximate method, Eq. (17), is in good agreement with that computed using the structural information, Eq. (13). The same is true for the stability boundary for torsional oscillations (Fig. 4).

Figures 3 and 4 are very instructive. They show that if the structure were stiffened, so the frequency increased by a factor of a certain magnitude, the damping can be decreased by the same factor and still give the same aeroelastic safety margin. This is fortunate, since when the structure is stiffened, it usually means that the contribution to the structural damping from sliding or deforming joints is lessened.

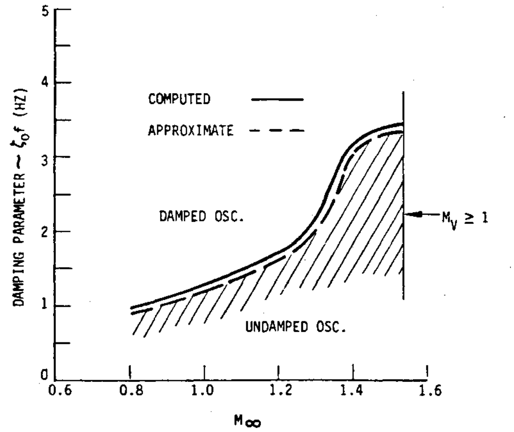

Fig. 3 Stability boundary for bending oscillations of the LO_2 cable tray.

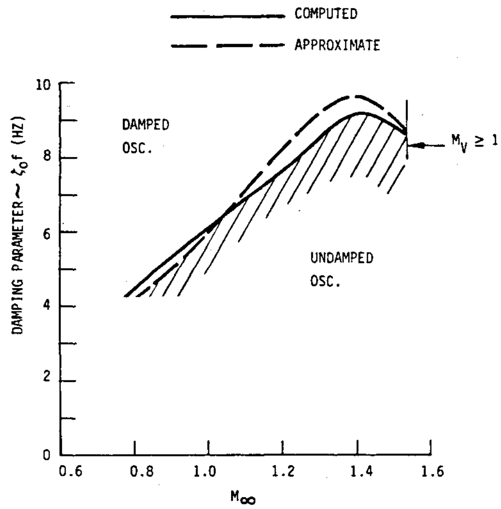

Fig. 4 Computed and approximated stability boundaries for torsional oscillations of the LO_2 cable tray.

Conclusions

A simple analysis has been presented showing how the flutter boundaries for beams can be determined with sufficient accuracy for preliminary design knowing only the beam natural frequency and structural damping. Through a chosen example from the analyses of the aeroelastic stability of Space Shuttle proturberances it has been illustrated how this simplified analysis technique can be used to quickly assess the effects of structural design changes, thereby facilitating the search for means whereby any eventual flutter problem may be eliminated.

References

[1] Ericsson, L.E. and Reding, J.P., "Aeroelastic Analysis of the Space Shuttle External Tank Cable Trays," LMSC D766543, Contract ASO-751485, Lockheed Missiles & Space Company, Inc., Sunnyvale, Calif., April 1981.

[2] Ericsson, L.E. and Reding, J.P., "Aeroelastic Stability of Space Shuttle Protuberances," *Journal of Spacecraft and Rockets,* Vol. 19, July-Aug. 1982, pp. 307-313.